



# **DIGITAL SIGNATURE**

**INTRODUCTION, CLASSIFICATION, DELEGATION**

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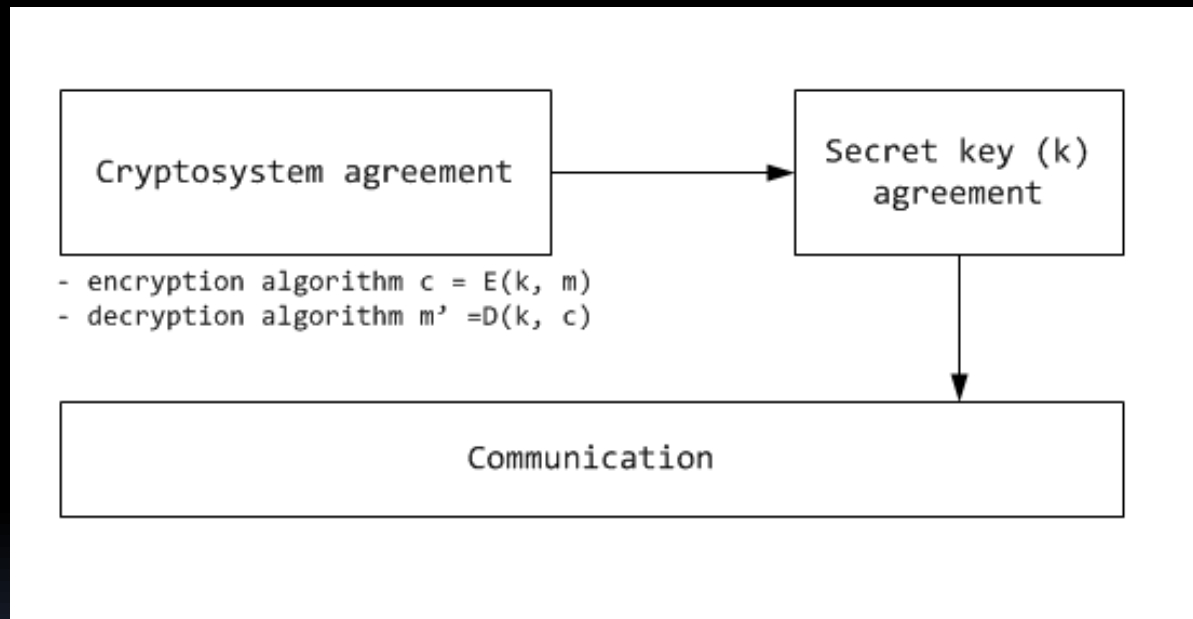
# ABSTRACT

- Conventional cryptography
- Diffie – Hellman concept
- Man-in-the-middle attack
- Public key cryptography
- Hash functions
- Digital signature
  - Basic schemes
  - Classification
  - Delegated signature schemes

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# Conventional cryptography



# Conventional cryptography

- Key must be transmitted by means of a secure channel (courier/meeting)
- If compromised – key may be misused (decryption of real messages, encryption of false messages, etc.)
- There's no way to conclude from the ciphertext who was the sender (Bob can send message to himself) – source of forgery
- Key management –  $n(n-1)/2$

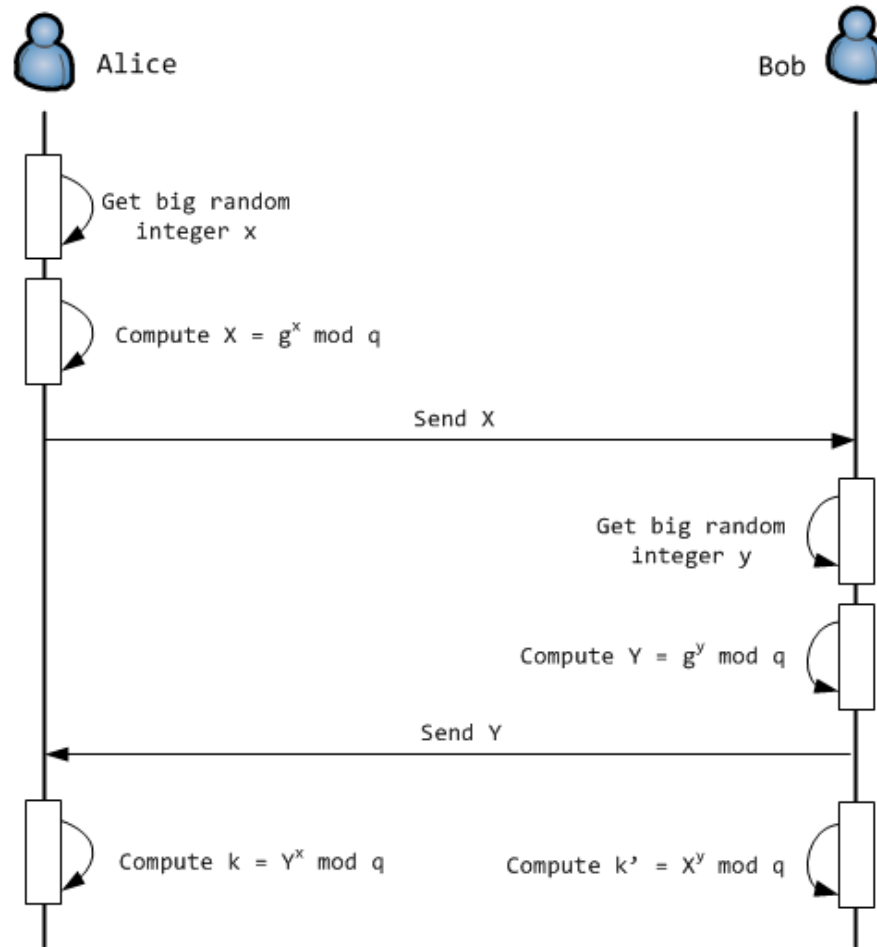
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# Diffie-Hellman concept

- Exponential key agreement protocol
- First known public key algorithm
- Cryptosystem
  - $q$  – power of a prime number, defines the order of the finite field  $F_q$
  - $g$  – generator of the multiplicative group of order  $q-1$
- Security based on discrete logarithm problem

# Diffie-Hellman concept



$$k = Y^x \text{ mod } q = g^{xy} \text{ mod } q = X^y \text{ mod } q = k'$$





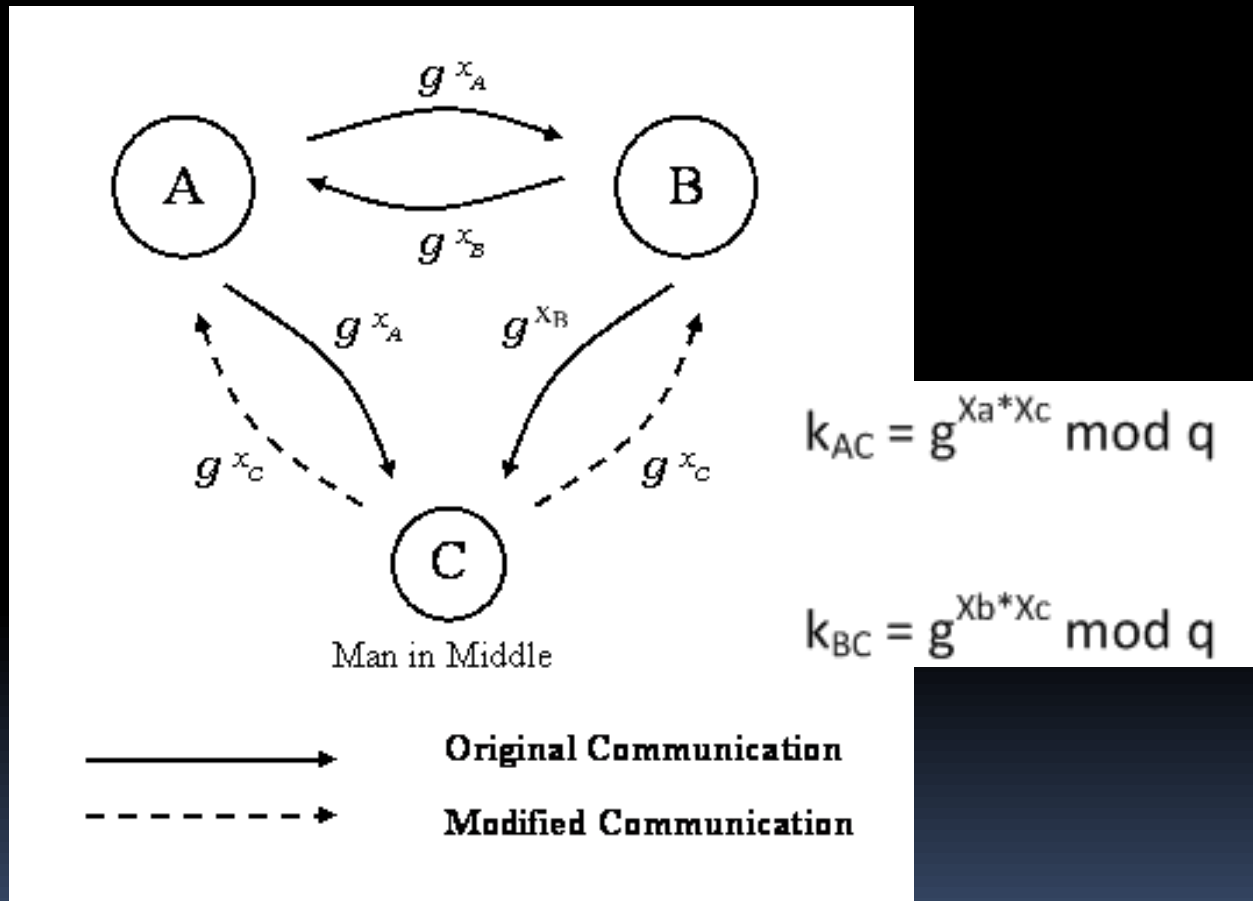
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# Man-in-the-middle attack

- DH cannot stand against MIM attack
- Intruder Mallory may interrupt the communication during key exchange
- Cryptosystem = DH cryptosystem
  - $p, q, F_q$

# Man-in-the-middle attack

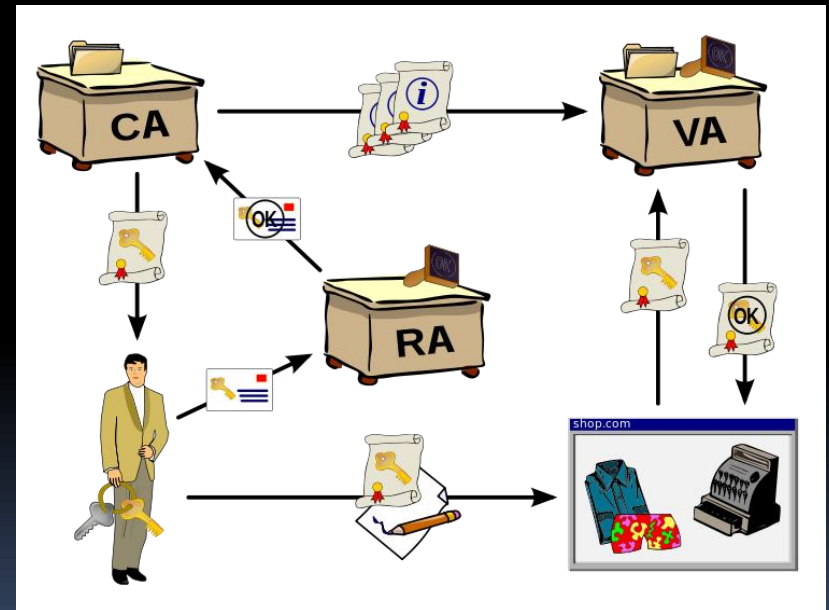


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# Public key cryptography

- Success of the MIM attack – sides cannot be sure whose key they are using
- Remedy?
  - Trusted authority
  - Digital certificate





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# Hash functions

- Mapping any length messages to fixed length message digests
- $h = H(M)$ 
  - $h$  – message digest
  - $M$  – message of any length
- Features
  - Having  $M$  – it is easy to compute  $h = H(M)$
  - One way property
  - Collision resistance property



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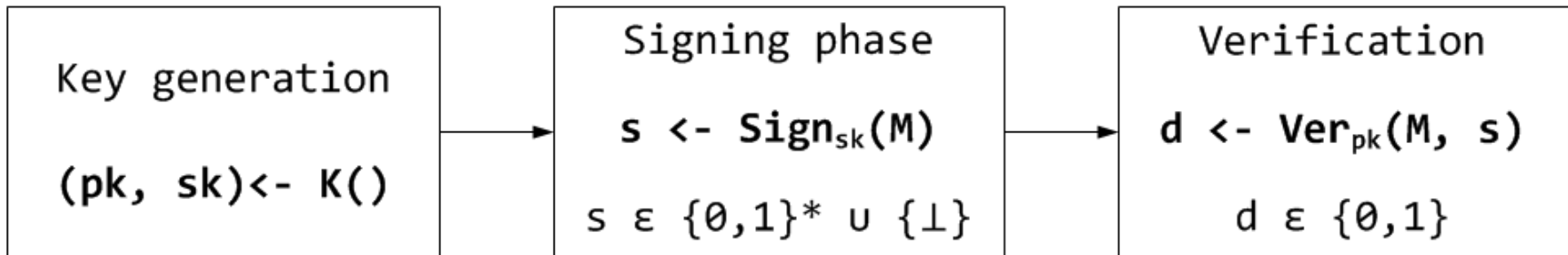


# Digital signature

- Electronic analogue to hand written signature
- Provided security services:
  - Authentication
  - Non-repudiation
  - Integrity
- Messages with signature can be encrypted and time-stamped

# Digital signature

- Digital signature scheme:
  - The base of every signature algorithm
  - Consists of three phases



# Digital signature - RSA

- Rivest, Shamir, Adleman
- Security based on the IF problem

## Key generation

- generate two large distinct random primes  $p$  and  $q$
- compute  $n = p \cdot q$  and Euler function  $\varphi(n) = (p-1)(q-1)$
- select a random integer  $e$ , so that  $1 < e < \varphi(n)$ , such that  $\gcd(e, \varphi(n)) = 1$
- compute  $d$ , so that  $1 < d < \varphi(n)$ , such that  $ed \equiv 1 \pmod{\varphi(n)}$ . In other words,  $d = e^{-1} \pmod{\varphi(n)}$  is the inverse of  $e$ .
- $(e, n)$  - public key
- $(d, n)$  - private key
- message space  $M$  and ciphertext space  $C$  is  $Z_n = \{0, 1, 2, \dots, n-1\}$

# Digital signature - RSA

## Signature generation

- uses private key
- compute signature  $s = m^d \bmod n$

## Signature verification

- uses signer's public key  $(e, n)$
- compute  $m' = s^e \bmod n = m^{ed} \bmod n$
- verify if  $m' = m$

# Digital signature - RSA

## Proof

- let's remind the Euler theorem:
  - if  $a$  and  $n$  are relatively prime, then  $a^{\varphi(n)} = 1 \pmod n$
- moreover:  $ed \equiv 1 \pmod{\varphi(n)} \Rightarrow ed = 1 + x * \varphi(n)$
- so:  $m^{ed} = m * m^{x*\varphi(n)} = m * 1^x = m \pmod n$

# Digital signature - ElGamal

- Security based on the DL problem

## Key generation

- Take a large prime number  $p$  defining a finite field  $Z_p$
- Find  $g$  - generator of a multiplicative group  $Z^*_p$
- compute random secret number  $x$ ,  $1 < x < p$
- compute  $y = g^x \text{ mod } p$
- $\{p, g, y\}$  - public key
- $x$  - private key

# Digital signature - ElGamal

## Signature generation

- uses private key  $x$ , and cryptosystem parameters  $(p, g)$
- select a random integer  $k$ ,  $1 < k < p-1$  such that  $\gcd(k, p-1) = 1$
- compute  $r = g^k \bmod p$  and  $k^{-1} \bmod (p-1)$
- compute  $s = k^{-1} [m - x*r] \bmod (p-1)$
- $(r, s, m)$  - signature

## Signature verification

- uses public key  $\{p, g, y\}$
- check if  $1 < r < (p-1)$
- compute  $v_1 = y^r * r^s \bmod p$
- compute  $v_2 = g^m \bmod p$
- the signature is valid if and only if  $v_1 = v_2$

# Digital signature - ElGamal

**Proof**

$$\begin{aligned}v_1 &= y^r * r^s \text{ mod } p = g^{x*r} * r^{k^{-1}*[m-x*r]} \text{ mod } p \\ &= g^{x*r} * g^{k * k^{-1}*[m-x*r]} \text{ mod } p = g^m \text{ mod } p = v_2\end{aligned}$$



# Signature classification

1) By mathematical problem on which their security is based

- Based on IF problem
- Based on DL problem

2) By the signer identification method

- Certificate - based signatures
- ID-based signatures

# Signature classification

## 3) By the usage of randomization

- randomized schemes
- deterministic schemes

## 4) By the ability to recover message

- Schemes with appendix
- TMR (Total Message Recovery)
- PMR (Partial Message Recovery)

# Signature classification

- Signature schemes may belong to many of presented groups
- Most of them have special features – another way of classification
- They still use RSA or ElGamal signature scheme

# Blind signatures

- Introduced by D. Chaum
- The signer knows neither the message nor its signature
- Alice generates blinding function  $m' = f(m)$  and sends to BOB
- Bob signs  $s' = \text{Sign}(m')$  and sends the result back to Alice
- Alice computes the reverse blinding function  $f'(s') = s$  and gets the signature
- Usage: e-money, e-voting

# Undeniable signatures

- Proposed by David Chaum and Hans van Antwerpen
- Digital signatures can be copied exactly
- Verification is possible only with the interaction with the signer
  - only authorized entities can access the document to verify the signature
  - signer cannot deny a valid signature (disavowal protocol)

# Designated Confirmer Signatures

- Compromise between self-authenticating signatures and undeniable signatures
- A designated confirmer allows certain designated parties to confirm the authenticity at any time without asking signer
- Others are not able to verify signature without the aid of designated parties or the signer himself

# Directed Signatures

- Proposed by Lim and Lee
- Self-authentication property is not suitable for applications like signing personal information (tax bills, prescriptions, etc.)
- Signer sends signed message  $m$  to the designated verifier (i.e. patient) while others know nothing on the origin and validity of the message without help of signer or the designated verifier
- Both signer and designated verifier can prove to any third party that the signature is valid

# Nominative Signatures

- Includes two parties:
  - nominator - generates a digital signature
  - nominee - verifies the validity
- Only nominee can verify the nominator's signature
- Only nominee can prove to some third party that the signature is issued to him and is valid



# Group Signatures

- Assume we have a group of users
  - every member is authorized to sign documents on behalf of the group
  - The signature generated by any member is called a group signature
- The receiver of the signature:
  - can verify if it represents the particular group
  - cannot identify which member signed it
- Group members or TA can identify the signer

# Ring signatures

- User from the set:
  - can convince the verifier that the signer belongs to the set
  - cannot identify signer
- Unlike group signature, requires additional setup:
  - group manager
  - setup procedure
  - action of the non-signing members
- For signing purposes, signer may choose random set of other possible signers including himself

# Threshold signatures

- $(t, n)$  threshold scheme
  - a secret key  $k$  is shared among  $n$  members of a group
  - any  $t$  members are able to cooperate and reconstruct the key  $k$
- $(t-1)$  or less users cannot reveal anything about the key
- But any set of  $t$  or more shareholders can impersonate any other set.
- Malicious set of signers does not have any responsibility for the signatures

# Multi Signatures

- Used in applications that require require the signature of more than one person (e.g. bank account, government, etc.)
- Signing is possible only if multiple keys are available
- Each signer produces a valid partial signature on message which is combined further to get a complete signature

# Proxy Signatures

- Delegated signature schemes
- Enables original signer to delegate signing authority to a proxy signer
  - temporal absence
  - lack of time or computational power
- Proxy signer can compute a signature, that can be verified with the original signer's public key

# Full Delegation

- Alice gives her private key to Bob
- Bob using Alice's private key computes signature
- such signature is indistinguishable from the normal signature

# Partial Delegation

- Alice computes proxy key  $s$  from her private key  $x$  and gives it to the proxy signer Bob in a secure way
- Bob computes signature with the key  $s$
- Such signature is distinguishable from the original signature
- For security reasons key  $x$  should not be computable from  $s$

# Delegation by Warrant

- Warrant - certificate composed of:
  - a message part (that the proxy signer is authorized to sign)
  - public key
- Delegate proxy
  - Alice signs a warrant and declares Peter as proxy signer
  - To sign message, Peter simply signs it and combines with the warrant
  - Warrant differentiates between Peter's normal signature and the proxy signature
- Bearer proxy
  - Alice computes proxy key pair, signs a warrant, and gives to Peter



# Partial Delegation with Warrant

- Compromise between delegation by warrant and partial delegation
- Alice generates a secret  $s$  from her private key, and includes a warrant
- Alice sends the secret to Peter in a secure way

# Threshold Delegation

- Designed for group oriented societies
- $(t, n)$  threshold delegation
  - Alice distributes proxy signature key among  $n$  proxy signers
  - To generate a valid proxy signature we need at least  $t$  signatures ( $t \leq n$ )



**THANK YOU!**