

# DIGITAL SIGNATURE DELEGATION

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# AGENDA

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- ▶ DIGITAL SIGNATURE DELEGATION
  - ▶ GENERAL DEFINITION
  - ▶ MATHEMATICAL DESCRIPTION
  - ▶ CLASSIFICATION
  - ▶ SECURITY REQUIREMENTS
  - ▶ ALGORITHMS
    - ▶ SCHNORR SIGNATURE SCHEME
    - ▶ MUO SCHEME
    - ▶ ZHANG SCHEME

# GENERAL DEFINITION

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- ▶ Enables original signer to delegate signing authority to a proxy signer
  - ▶ Temporal absence
  - ▶ Lack of time or computational power
- ▶ Proxy signer can compute a valid signature, that can be verified with the original signer's public key

# MATHEMATICAL DEFINITION

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$$PS = (\underbrace{G, \mathcal{K}, S, \mathcal{V}}_{DS}, (D, P), PS, P\mathcal{V}, ID)$$

DS =  $(G, \mathcal{K}, S, \mathcal{V})$  - digital signature scheme, where

- $G$  - Randomized parameter-generation algorithm, which takes input  $1^\kappa$ , where  $\kappa$  is the security parameter, and outputs some global public params
- $\mathcal{K}$  - Key generation algorithm - takes input global parameters and outputs a pair  $(pk, sk)$
- $S$  - Signature algorithm - takes input a secret key  $sk$  and a message  $M \in \{0,1\}^*$  and outputs a signature  $\sigma$
- $\mathcal{V}$  - Deterministic verification algorithm - takes input a public key  $pk$ , a message  $M$ , and a candidate signature  $\sigma$ , and outputs a bit (1 if signature is valid, 0 - otherwise)

# MATHEMATICAL DEFINITION

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$$PS = (\mathcal{G}, \mathcal{K}, S, \mathcal{V}, \underbrace{(\mathcal{D}, \mathcal{P})}, PS, \mathcal{PV}, ID)$$

$(\mathcal{D}, \mathcal{P})$  - a pair of randomized algorithms forming proxy-designation protocol

$$sk_p \leftarrow [\mathcal{D}(pk_i, sk_i, j, pk_j, \omega), \mathcal{P}(pk_j, sk_j, pk_i)]$$

- $\mathcal{D}$  - takes input designator  $i$  public key  $pk_i$ , proxy signer  $j$  public key  $pk_j$ , the identity  $j$  of the proxy signer, and a message space descriptor  $\omega$  for which user  $i$  wants to delegate its signing rights. Returns no output
- $\mathcal{P}$  - takes input designator  $i$  public key  $pk_i$ , proxy signer  $j$  public and private keys  $pk_j, sk_j$  and produces  $sk_p$  - proxy signing key, that user  $j$  uses to produce proxy signatures on behalf of user  $i$

# MATHEMATICAL DEFINITION

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$$PS = (\mathcal{G}, \mathcal{K}, S, \mathcal{V}, (\mathcal{D}, \mathcal{P}), \underbrace{PS}_{\text{}}, \mathcal{PV}, ID)$$

( $PS$ ) - (possibly) randomized proxy signing algorithm, that takes input a proxy signing  $skp$  and a message  $\mathcal{M} \in \{0,1\}^*$ , and outputs a proxy signature  $p_\sigma \in \{0,1\}^* \cup \{\perp\}$

$$p_\sigma \leftarrow PS(skp, \mathcal{M})$$

# MATHEMATICAL DEFINITION

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$$PS = (\mathcal{G}, \mathcal{K}, S, \mathcal{V}, (\mathcal{D}, \mathcal{P}), PS, \underbrace{\mathcal{PV}}, ID)$$

$(\mathcal{PV})$  - deterministic proxy verification algorithm, that takes input a public key  $pk$  (corresponding to proxy secret key  $skp$ ), a message  $M \in \{0,1\}^*$ , and a proxy signature  $p\sigma \in \{0,1\}^* \cup \{\perp\}$  and outputs 0 or 1. and outputs a proxy signature

$$\{0, 1\} \leftarrow \mathcal{PV}(pk, M, p\sigma)$$

# MATHEMATICAL DEFINITION

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$$PS = (\mathcal{G}, \mathcal{K}, S, \mathcal{V}, (\mathcal{D}, \mathcal{P}), \mathcal{PS}, \mathcal{PV}, \underbrace{ID})$$

$(ID)$  - proxy identification algorithm which takes input a valid proxy signature  $p_\sigma$  and outputs an identity  $i \in \mathcal{N}$  or  $\perp$  in case of error

$$\mathcal{N} \cup \{\perp\} \leftarrow ID(p_\sigma)$$



# CLASSIFICATION

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- ▶ Full delegation
- ▶ Partial delegation
  - ▶ Proxy-unprotected proxy signature
  - ▶ Proxy-protected proxy signature
- ▶ Delegation by warrant
  - ▶ Delegate proxy
  - ▶ Bearer proxy
- ▶ Partial delegation with warrant
- ▶ Threshold delegation

# DIGITAL SIGNATURE DELEGATION

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- ▶ Security requirements
  - ▶ Unforgeability
  - ▶ Proxy signer's deviation
  - ▶ Secret keys dependence
  - ▶ Verifiability
  - ▶ Distinguishability
  - ▶ Indentifiability
  - ▶ Undeniability

Only a designated proxy signer can generate a valid proxy signature

# DIGITAL SIGNATURE DELEGATION

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A proxy signer cannot generate a valid proxy signature which is not detected as generated by him

# DIGITAL SIGNATURE DELEGATION

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  - ▶ Undeniability

A proxy key should always be generated from the original signer's private key

# DIGITAL SIGNATURE DELEGATION

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- ▶ **Security requirements**
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  - ▶ Undeniability

From given proxy signature, a verifier can be sure, that the original siger agreed on the signed message

# DIGITAL SIGNATURE DELEGATION

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A proxy signature must be distinguishable from a normal signature of the original signer

# DIGITAL SIGNATURE DELEGATION

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  - ▶ **Indentifiability**
  - ▶ Undeniability

The original signer can conclude from a proxy signature who signed the message

# DIGITAL SIGNATURE DELEGATION

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  - ▶ Secret keys dependence
  - ▶ Verifiability
  - ▶ Distinguishability
  - ▶ Indentifiability
  - ▶ **Undeniability**

A proxy signer cannot disavow a proxy signature generated by him



# SCHNORR SIGNATURE SCHEME

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## ▶ Security based on DL problem

### Key generation

- Take a large prime number  $p$  defining a finite field  $Z_p$
- Find  $g$  - generator of a multiplicative group  $Z_p^*$
- compute random secret number  $x$ ,  $1 < x < p$
- compute  $y = g^x \bmod p$
- $\{p, g, y\}$  - public key
- $x$  - private key

# SCHNORR SIGNATURE SCHEME

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## Signature generation

- uses private key  $x$ , and cryptosystem parameters  $(p, g)$
- select a random integer  $k$ ,  $1 < k < p-1$  such that  $\gcd(k, q-1) = 1$
- compute  $r = g^k \bmod p$
- compute  $e = h(m || r)$
- compute  $s = xe + k \bmod q$
- $(s, e)$  - signature

## Signature verification

- uses public key  $\{p, g, y\}$
- compute  $e' = h(m || g^s y^{-e} \bmod p)$
- the signature is valid if and only if  $e = e'$

# SCHNORR SIGNATURE SCHEME

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**Proof**

$$g^s y^{-e} \bmod p = g^{xe + k} g^{-xe} \bmod p = g^k \bmod p = r$$

# MUO SIGNATURE SCHEME

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- ▶ Proposed by Mambo, Usuda and Okamoto
- ▶ The scheme shows how to create proxy signature algorithms
- ▶ Allows to use any DL-based signature scheme to compute proxy signature

# MUO SIGNATURE SCHEME

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$p$  : a large prime number

$q$  : a prime factor of  $(p - 1)$

$g$  : an element of  $Z_p^*$  of order  $q$

$x$  : secret key of the original signer S, where  $x \in_R Z_q$

$y$  : public key of the original signer S, where  $y = g^x \pmod p$

# MUO SIGNATURE SCHEME

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## Proxy Generation

1. (Key Generation)- The original signer, Alice, selects  $k \in_R Z_q$ , and computes

$$r = g^k \bmod p \quad \text{and} \quad s = x + kr \bmod q$$

The original signer secretly sends  $(s, r)$  to the proxy signer.

2. (Key verification)- The proxy signer checks the validity of the key  $(s, r)$  by checking, if

$$g^s = yr^r \bmod p$$

He accepts  $(s, r)$  as his secret key iff  $(s, r)$  satisfies this congruence.

# MUO SIGNATURE SCHEME

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## Proxy Signing

When the proxy signer, Peter, signs a message  $m$  on behalf of the original signer, he computes a signature  $s_p$  using any original signature scheme (here we are using Schnorr scheme) and  $s$  as the secret key. Peter selects  $k_p \in_R (1, q - 1)$  and computes

$$r_p = g^{k_p} \bmod p \quad \text{and} \quad e = h(m || r_p)$$

$$s_p = se + k_p \bmod q$$

The pair  $(s_p, e)$  is Schnorr signature.  $(s_p, e, r)$  is the proxy signature.

## Verification

To verify the proxy signature the verifier, Bob, computes

$$e' = h(m || g^{s_p} (y r^r)^{-e} \bmod p)$$

He accepts the signature if and only if  $e' = e$ .

# MUO SIGNATURE SCHEME

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## Proof

$$\begin{aligned} g^{Sp}(yr^r)^{-e} \bmod p &= g^{Sp}(g^S)^{-e} \bmod p = \\ &= g^{se + kp} g^{-se} \bmod p = g^{kp} \bmod p = r_p \end{aligned}$$



# MUO SIGNATURE SCHEME

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- ▶ Proxy secret key is generated from original signer's private key
- ▶ Verifier can be convinced that signature comes from an authorised proxy signer
- ▶ Proxy signature is distinguishable from original signer's signature
- ▶ Knowledge of original signer's public key is sufficient to verify proxy signature

# MUO SIGNATURE SCHEME

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## ▶ Drawbacks

- ▶ Does not provide non-reputation – both users know the proxy secret key
- ▶ Requires secure channel to transport proxy key

## ▶ Remedy

- ▶ Kan Zhang's proxy key generation algorithm

# MUO SIGNATURE SCHEME

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## ▶ Drawbacks

- ▶ Does not provide non-reputation – both users know the proxy secret key
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# ZHANG'S ALGORITHM

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## 1. (Key Generation)-

(a) The original signer, Alice, selects a  $\bar{k} \in_R Z_q$ , and computes

$$\bar{r} = g^{\bar{k}} \bmod p$$

He sends  $\bar{r}$  to the proxy signer through a public channel.

(b) Proxy signer, Peter, selects  $\alpha \in_R Z_q$ , and computes

$$r = g^{\alpha \bar{r}} \bmod p$$

If  $r \in Z_q^*$ , he sends  $r$  to Alice, else repeats the process.

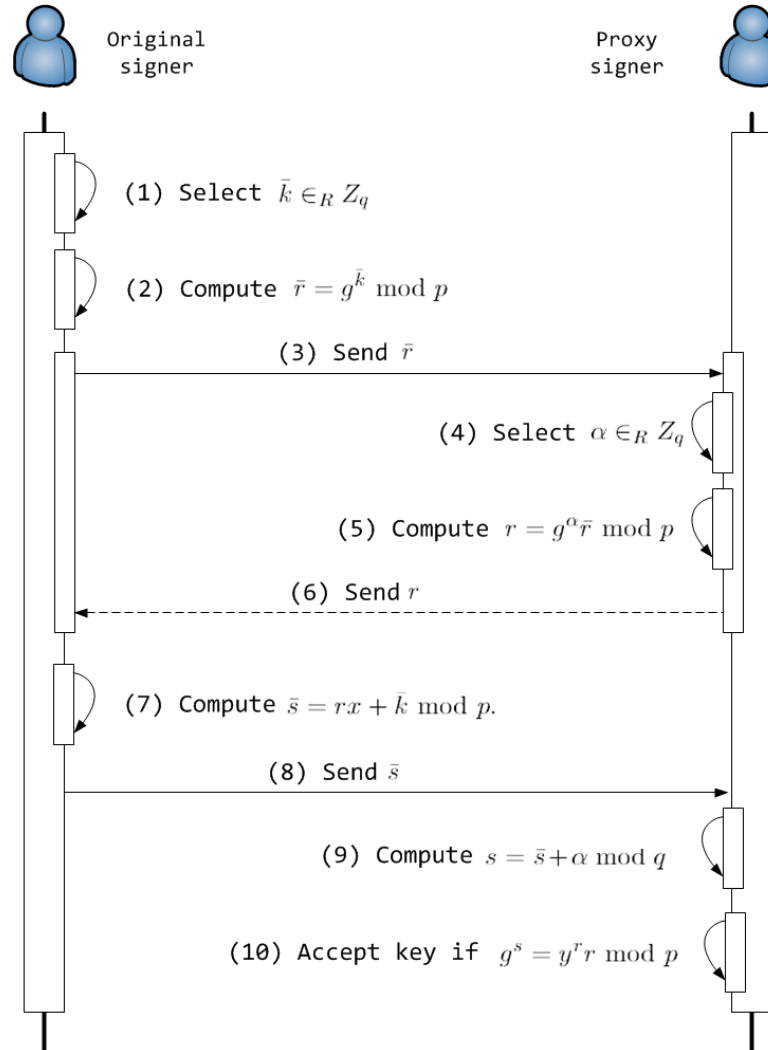
(c) Alice computes  $\bar{s} = rx + \bar{k} \bmod p$ .

## 2. (Proxy Key Delivery)- Alice sends $\bar{s}$ to the proxy signer, Peter.

3. (Key verification)- After receiving the  $\bar{s}$  the Peter modifies the key and obtain a new key  $s = \bar{s} + \alpha \bmod q$ . He accepts  $s$  as a valid proxy secret key iff  $(s, r)$  satisfies

$$g^s = y^r r \bmod p$$

# ZHANG'S ALGORITHM



# ZHANG'S ALGORITHM

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## Proof

$$\text{Left} = g^s \text{ mod } p = g^{\hat{s} + \alpha} \text{ mod } p = g^{rx + k + \alpha} \text{ mod } p$$

$$\text{Right} = y^r \text{ mod } p = g^{rx} g^{\alpha} r \text{ mod } p = g^{rx + \alpha + k} \text{ mod } p$$

# NEXT SEMINAR

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- ▶ More proxy signature algorithms
- ▶ Digital signature restrictions
- ▶ Combination of delegated signature schemes and restrictions



**THANK YOU!**

